

MELUHA INTERNATIONAL SCHOOL

INDIA

SR MPC

Time: 3 Hours

JEE MAINS MODEL

Date: 27-03-2020

SECTION – I

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

MATHS

SYLLABUS : Algebra of Matrices (up to adjoint of matrices), Determinants & Pair of straight lines & Limits and continuity

- The values of x , so that $\begin{bmatrix} 1 & x & 1 \\ 1 & x & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$ is /are
 - ± 2
 - 0
 - $\frac{-7 \pm \sqrt{35}}{2}$
 - $\frac{-9 \pm \sqrt{53}}{2}$
- Matrix X satisfying the equation $2X - Y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$, $2Y + X = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$ is
 - $2 \begin{bmatrix} -2 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$
 - $2 \begin{bmatrix} -2 & 1 & 1 \\ -1 & 2 & 0 \end{bmatrix}$
 - $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$
 - $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$
- If $A = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$, then $\det(AA^T)$ is equal to
 - 0
 - 7
 - 2
 - 3
- If $E(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then $E(\alpha) \cdot E(\beta)$ is equal to
 - $E(\alpha - \beta)$
 - $E(\alpha + \beta)$
 - $E(\alpha)$
 - $E(\beta)$
- If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ and $A(\text{adj } A) = \lambda I$, then the value of λ is
 - 1
 - 2
 - 3
 - 4
- If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then
 - $1 + \alpha^2 + \beta\gamma = 0$
 - $1 - \alpha^2 + \beta\gamma = 0$
 - $1 - \alpha^2 - \beta\gamma = 0$
 - $1 + \alpha^2 - \beta\gamma = 0$
- If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which of the following is correct?
 - $(A+B) \cdot (A-B) - A^2 - B^2$
 - $(A+B) \cdot (A-B) = A^2 - B^2$
 - $(A+B) \cdot (A-B) = 1$
 - None of these

17. The lines through origin which make angles of 45° with the line $lx + my + n = 0$ are

- 1) $(l^2 - m^2)(x^2 - y^2) + 4lmxy = 0$
- 2) $(l^2 + m^2)(x^2 - y^2) + 4lmxy = 0$
- 3) $(l^2 - m^2)(x^2 + y^2) + 4lmxy = 0$
- 4) $(l^2 - m^2)(x^2 + y^2) - 4lmxy = 0$

18. The equation of image of pair of lines $y = |x - 1|$ in y -axis is

- 1) $x^2 - y^2 + 5 = 0$
- 2) $x^2 + y^2 + 2x + 1 = 0$
- 3) $x^2 - y^2 + 2x + 1 = 0$
- 4) $x^2 - y^2 - 2x - 1 = 0$

19.
$$\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{3}{4^x}\right)}{\tan\left(\frac{4}{3^x}\right)} =$$

- 1) 0
- 2) 1
- 3) 2
- 4) 3

20.
$$\lim_{n \rightarrow \infty} \prod_{r=3}^n \left(\frac{r^3 - 8}{r^3 + 8} \right) =$$

- 1) $\frac{1}{7}$
- 2) $\frac{2}{7}$
- 3) $\frac{3}{4}$
- 4) $\frac{7}{2}$

SECTION-II

(Numerical Value Answer Type)

21. If $\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix} = kabc$, then the value of k is _____

22. If $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = k(x+y+z)^3$, then the value of k is _____

23. If $\Delta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then the value of $\Delta + \Delta_1$ is _____

24. If P_1, P_2 denote the length of the perpendiculars from the point $(2, 3)$ on the lines given by the equation $15x^2 + 31xy + 14y^2 = 0$ if $P_1 < P_2$ then $\frac{1}{5} \left(13P_1^2 + \frac{74}{31}P_2^2 \right) =$ _____

25. If the angle between the pair of straight lines $y^2 \sin^2 \theta - xy \sin^2 \theta + x^2 (\cos^2 \theta - 1) = 0$ is ϕ then the value of $5 \sin^2 \phi + 15 \sin \phi - 20 \cos^2 \phi =$ _____

SECTION – I

(SINGLE CORRECT ANSWER TYPE)

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PHYSICS

SYLLABUS: Impulse and Momentum, Work, Energy

26. Choose the correct alternative among the following:

- 1) When a conservative force does positive work on a body, the potential energy of the body increases
- 2) Work done by a body against friction always results in a loss of its kinetic energy
- 3) The rate of change of total momentum of a many particle system is proportional to the sum of the internal forces on the system
- 4) In an elastic collision kinetic energy of the colliding bodies before collision is equal to kinetic during collision

27. A spring of spring constant k is stretched by a length x from its natural length and released.

What about its potential energy:

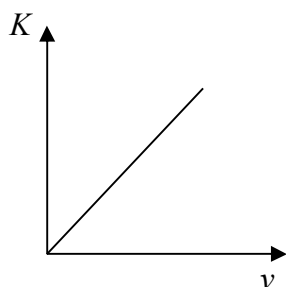
- 1) Remain stored in spring in any form
- 2) Converted into heat
- 3) Transfer to air molecules
- 4) None of the above

28. A light and a heavy body have equal momenta. Which one has greater K.E.?

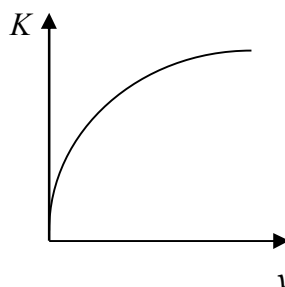
- 1) The light body
- 2) The heavy body
- 3) The K.E. are equal
- 4) data is incomplete

29. The graph of kinetic energy (K) of a body versus velocity (v) is represented as:

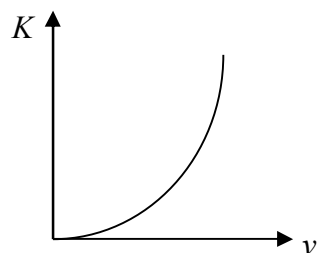
1)



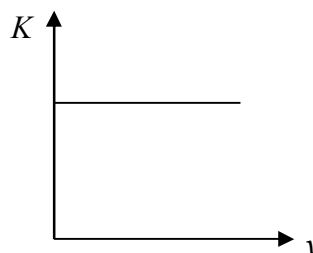
2)



3)



4)



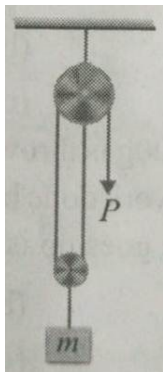
30. The only force acting on a 2kg body as it moves along the positive x axis has component $F_x = -6x$ N, where x is in metre. The velocity of the body at $x = 3$ m is 8 m/s. The velocity of the body at $x = 4$ m is:

- 1) 10m/s
- 2) 9 m/s
- 3) 7.4 m/s
- 4) 6.6 m/s

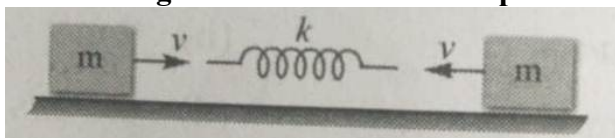
31. A toy gun has a spring of force constant k . After charging the spring by compressing it through a distance x , the toy releases off shot of mass m vertically upwards. Then the shot will travel a vertical height of

- 1) $\frac{2mg}{kx}$
- 2) $\frac{kx^2}{mg}$
- 3) $\frac{kx}{mg}$
- 4) $\frac{kx^2}{2mg}$

32. The body of mass m is pulling, vertically up with constant speed, by applying force P . the free end of the string is pulled by l meter, the increase in potential energy of the block is:



- 1) $\frac{mgl}{2}$ 2) mgl 3) $2mgl$ 4) $\frac{mgl}{4}$
33. A block of mass M is through over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F . the kinetic energy of the block increases by 20 J in 1 s:
- 1) The tension of the string is Mg
 2) The tension in the string is F .
 3) The work done by the tension on the block is 20 J in the above 1 s.
 4) The work done by the force of gravity is -20 J in the above 1 s.
34. A force acts on a 30g particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is
- 1) 5.28 J 2) 450 J 3) 490 J 4) 530 J
35. A particle of mass 100g is thrown vertically upwards with a speed of 5m/s. The work done by the force of gravity during the time particle goes up is
- 1) -1.25 J 2) 5.17 kJ 3) 0.5 J 4) -0.5 J
36. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100m. it slides down a smooth surface to the ground, them climb up another hill of height 30m and finally slides down to a horizontal base at a height of 20m above the ground. the velocity attained by the ball is
- 1) $10m/s$ 2) $10\sqrt{30}m/s$ 3) $40m/s$ 4) $20m/s$
37. Two blocks, each of mass m moving with speed v collide with the spring of force constant k as shown in figure. The maximum compression of the spring is:



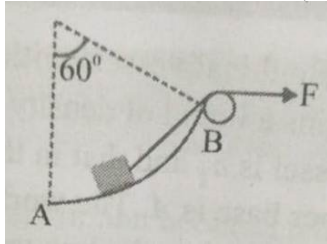
- 1) $\sqrt{\frac{mv^2}{k}}$ 2) $\sqrt{\frac{2mv^2}{k}}$ 3) $\sqrt{\frac{mv^2}{2k}}$ 4) Zero
38. Calculate the work done, if a wire is loaded by ' Mg ' weight and the increase in length is ' l ':
- 1) Mgl 2) $\frac{Mgl}{2}$ 3) $2Mg l$ 4) Zero
39. A 60g bullet is fired through a stack of fibre board sheers 200mm thick. If the bullet approaches the stack with a velocity of 600m/s, and emerges out with a velocity of 300 m/s, the average resistance offered to the bullet is:
- 1) 40.5 kN 2) 2 k N 3) 20.25 kN 4) 10 kN
40. A particle in a conservative force field has a potential energy given by $U = \left(\frac{20xy}{z}\right)$. The force exerted on it is

- 1) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$ 2) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} + \left(\frac{20xy}{z^2}\right)\hat{k}$
 3) $-\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$ 4) $\left(\frac{20y}{z}\right)\hat{i} + \left(\frac{20x}{z}\right)\hat{j} - \left(\frac{20xy}{z^2}\right)\hat{k}$

41. A locomotive of mass m starts moving so that its velocity varies according to law $v = a\sqrt{s}$, where a is a constant, and s is the distance covered. The total work done performed by all the forces which are acting on the locomotive during first t seconds after the beginning of motion is:

- 1) ma^2t 2) $\frac{ma^4t}{2}$ 3) $\frac{ma^4t^2}{8}$ 4) None of these

42. A 10 kg block is pulled in the vertical plane along a frictionless surface in the form of an arc of a circle of radius 10m. the applied force is of 200N as shown in figure. If the block had started from rest at A, the velocity at B would be:



- 1) 1.7 m/s 2) 17 m/s 3) 27 m/s 4) 34 m/s

43. A ball of mass 'm' is thrown at an angle is ' θ ' with the horizontal with an initial velocity 'u'. The change in its momentum during its flight in a time interval of 't' is

- 1) mgt 2) $mgt \cos \theta$ 3) $mgt \sin \theta$ 4) $\frac{1}{2}mgt$

44. An impulse "I" given to a body change its velocity from " v_1 to v_2 ". The increase in the kinetic energy of the body is given by

- 1) $I(v_1 + v_2)$ 2) $I(v_1 + v_2)/2$ 3) $I(v_1 - v_2)$ 4) $I(v_1 - v_2)/2$

45. An impulse is supplied to a moving object with the force at an angle 120° with the velocity and the change in momentum vector is

- 1) 120° 2) 0° 3) 60° 4) 240°

SECTION- II

(Numerical Value Answer Type)

46. A force $F = 6\vec{i} + 2\vec{j} + 3\vec{k}$ acts on a particle and produces a displacement of $\vec{s} = 2\vec{i} - 3\vec{j} - x\vec{k}$. If the work done is zero, the value of x is: _____
47. A body of mass 0.5kg travels in a straight line with velocity $v = 5x^{3/2}$. The work done by the net force during the displacement from $x = 0$ to $x = 2m$ is (J) _____
48. Figure gives the acceleration of a , 2.0 kg body as it moves from rest along x -axis while a variable force acts on it from $x = 0$ m rest to $x = 9m$. The work done by the force on the body when it reaches $x = 7m$ is given by (J) _____
49. At $t = 0$ a 1.0 kg ball is thrown from the top of a tall tower with velocity $\vec{v} = (8\hat{i} + 9\hat{j}) m/s$. The change in the potential energy of the ball – earth system between $t = 0$ to $t = 5 s$:
 ($g = 10m/s^2$) (J) _____
50. A body is acted on by a force given by $F = (10 + 2t) N$. The impulse received by the body during the first four seconds is (Ns) _____

62. Calculate the amount of MgS formed when 2g of Mg reacts with 2g of S
 1) 4.5 g 2) 3.5 g 3) 2.5 g 4) 0.5 g
63. Addition of Zinc powder to $CuSO_4$ solution precipitates copper due to
 1) reduction of Cu^{2+} 2) reduction of SO_4^{2-}
 3) reduction of Zn 4) hydrolysis of $CuSO_4$
64. Same amount of metal carbines with 0.1 g of oxygen and 1g of a halogen. Hence the equivalent mass of halogen is
 1) 9 2) 35.5 3) 80 4) 127
65. An oxide of sulphur contains 50% of sulphur. What will be its empirical formula?
 1) SO 2) SO_2 3) SO_3 4) S_2O_3
66. A solution contains one mole of alcohol and four moles of water. What are the fractions of water and alcohol?
 1) 1/4, 4/1 2) 4/1, 1/4 3) 4/5, 1/5 4) 1/5, 4/5
67. Li_2O is one of the most efficient absorbents for CO_2 in space crafts in terms of absorbing capacity per unit mass. If the reaction is $Li_2O + CO_2 \rightarrow Li_2CO_3$. What if the absorption efficiency of pure Li_2O in litre CO_2 (STP) per kg
 1) 746.66 L/kg 2) 7466.6 L/kg 3) 74.66 L/kg 4) None of these
68. What is the charge involved when 0.1 mol of $C_6H_5NO_2$ is reduced to C_6H_5NHOH
 1) 0.3 F 2) 0.4 F 3) 0.6 F 4) 0.8 F
69. A gaseous hydro carbon gives upon combustion 0.72g of water and 3.08 g of CO_2 . The empirical formula of the hydrocarbon is
 1) C_3H_4 2) C_6H_5 3) C_7H_8 4) C_2H_4
70. The volume strength of 1.15 N H_2SO_4 solution is
 1) 4.8 2) 8.4 3) 3.0 4) 8.0

SECTION- II**(Numerical Value Answer Type)**

71. The density of a gaseous element is 5 times that of oxygen under similar condition. If the molecule of the element is Triatomic. What will be its atomic mass?
72. What volume of 75% alcohol by mass ($d = 0.8 \text{ g/cm}^3$) must be used to prepare 150 cc of 30% alcohol by mass ($d = 0.9 \text{ g/cm}^3$)?
73. Given that the abundances of isotopes ^{54}Fe , ^{56}Fe and ^{57}Fe are 50%, 90% and 5% respectively the atomic mass of Fe is
74. A saturated solution is prepared at $70^\circ C$ containing 32.0 g $CuSO_4 \cdot 5H_2O$ per 100 g solution. A 335 g sample of this solution is then cooled to $0^\circ C$ so that $CuSO_4 \cdot 5H_2O$ crystallizes out. If the concentration of saturated solution at $0^\circ C$ is 12.5 g $CuSO_4 \cdot 5H_2O$ per 100 g solution. How much $CuSO_4 \cdot 5H_2O$ is crystallized.
75. How many grams $KMnO_4$ are contained in 4 litres of 0.05 N solution the $KMnO_4$ is to be used as an oxidant in acid medium

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KEY SHEET**MATHS**

1) 4	2) 3	3) 1	4) 2	5) 1	6) 3	7) 2	8) 3	9) 4	10) 2
11) 1	12) 2	13) 4	14) 2	15) 4	16) 2	17) 1	18) 3	19) 1	20) 2
21) 4	22) 2	23) 0	24) 35	25) 20					

PHYSICS

26) 2	27) 3	28) 1	29) 3	30) 4	31) 4	32) 1	33) 2	34) 1	35) 1
36) 3	37) 2	38) 2	39) 1	40) 2	41) 3	42) 2	43) 1	44) 2	45) 2
46) 2	47) 50	48) 30	49) 800	50) 56					

CHEMISTRY

51) 1	52) 1	53) 3	54) 4	55) 4	56) 4	57) 1	58) 4	59) 1	60) 3
61) 1	62) 2	63) 1	64) 3	65) 2	66) 3	67) 1	68) 2	69) 3	70) 2
71) 53.33	72) 67.5	73) 55.95	74) 74.47	75) 6.32					

HINTS & SOLUTIONS

MATHS - A

1. Take $\begin{bmatrix} 1 & x & 1 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$

$$\begin{bmatrix} 1 & 5x+6 & x+4 \\ 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$$

$$1 + 5x + 6 + x^2 + 4x = 0$$

$$x^2 + 9x + 7 = 0$$

$$x = \frac{-9 \pm \sqrt{53}}{2}$$

2. **Given** $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$

$$2x - y = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & 4 \end{bmatrix}$$

Multiply 2 with $4 \times -2y = \begin{bmatrix} 6 & -6 & 0 \\ 6 & 6 & 4 \end{bmatrix}$

$$5X = \begin{bmatrix} 10 & -5 & 5 \\ 5 & 10 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

3. Since, $A = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix}$, and $A^T = \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$

$$AA^T = \begin{bmatrix} a & p \\ b & q \\ c & r \end{bmatrix} \begin{bmatrix} a & b & c \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + p^2 & ab + pq & ac + pr \\ ab + qp & b^2 + q^2 & bc + qr \\ ac + pr & bc + qr & c^2 + r^2 \end{bmatrix}$$

4. Since, $E(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$$E(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

and $E(\beta) = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$

$$E(\alpha).E(\beta) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos \alpha \cos \beta - \sin \alpha \sin \beta & \cos \alpha \sin \beta + \sin \alpha \cos \beta \\ -\sin \alpha \cos \beta - \cos \alpha \sin \beta & -\sin \alpha \sin \beta + \cos \alpha \cos \beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ -\sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix}$$

$$\therefore E(\alpha).E(\beta) = E(\alpha + \beta)$$

5. Since, $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

$$\text{Adj}(A) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\therefore A.(adjA) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & -\sin \theta \cos \theta + \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A.(adjA) = 1$$

But $\Rightarrow A.(adjA) = \lambda 1$

$$\Rightarrow \lambda = 1$$

6. $AA = 1$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \gamma\alpha & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$\alpha^2 + \beta\gamma = 1 \Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 1$$

7. $A + B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$

$$A - B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

and

$$B^2 = B.B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

and

$$(A+B)(A-B) = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$$

hence, $(A+B)(A-B) \neq A^2 - B^2$

$$8. \quad \cos \alpha = \frac{1 - \tan^2\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{1-t^2}{1+t^2}$$

$$\sin \alpha = \frac{2 \tan\left(\frac{\alpha}{2}\right)}{1 + \tan^2\left(\frac{\alpha}{2}\right)} = \frac{2t}{1+t^2}$$

$$= \begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{1+t^2}{1+t^2} \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & -t+0 \\ t+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

On putting the value of t in both equations, we get

$$\begin{bmatrix} 1 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\left(\frac{\alpha}{2}\right) \\ \tan\left(\frac{\alpha}{2}\right) & 1 \end{bmatrix}$$

$$\therefore (I-A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} = I+A$$

$$9. \quad 2x+3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$3x+2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and then subtracting, we get

$$\Rightarrow 4x+6y-9x-6y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3} \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

$$10. \quad A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$Q_1 = \frac{1}{2} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$Q_2 = \frac{1}{2}(A-A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$$Q_1 Q_2 = Q_3$$

$$11. \quad 2x^2 + 5xy + 2y^2 = (2x-y)(x-2y)$$

G divides \overline{OD} in 2:1

$$\Rightarrow D = \left(\frac{3}{2}, \frac{3}{2}\right)$$

Slope of $OD = 1 = m$

$OD \perp AB \Rightarrow$ equation of AB is

$$y - \frac{3}{2} = \frac{-1}{1} \left(x - \frac{3}{2}\right)$$

$$\Rightarrow x + y = 3$$

$$\Rightarrow x + y - 3 = 0$$

12. Eliminate 'x' in two equations

$$\Rightarrow (ag_1 - a_1g)x^2 + (2hg_1 - 2h_1g)xy + (bg_1 - b_1g)y^2 = 0$$

and $a+b=0$

$$(Coeff \text{ of } x^2 + Coeff \text{ of } y^2 = 0)$$

13. We have $h = \lambda, g = K, c = 2K, f = K$

Apply $\Delta = 0$, we get a quadratic equation in λ for real $\lambda \Rightarrow b^2 - 4ac \geq 0$

14. Let $m_1 m_2 m_3 = \frac{-a}{d}$ and $y = \frac{a}{d}x$ in the given equation

15. $\tan\theta = \sqrt{3}$, the line is $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$

$$\Rightarrow \frac{x-0}{1} = \frac{y-0}{\sqrt{3}} = r \text{ any point on the}$$

Line is $\left(\frac{r}{2}, \frac{\sqrt{3}}{2}r\right)$, where 'r' is the distance

from (0,0)

Substituting in the curve

$$r^3 \left(\frac{1+3\sqrt{3}}{8}\right) + r^2(\dots) + r(\dots) - 1 = 0$$

It is a cubic equation in 'r'

$$\therefore (OA)(OB)(OC) = \frac{1}{(1+3\sqrt{3})} (8) = \frac{4}{13} (3\sqrt{3}-1)$$

16. $S = (l_1x + m_1y + n_1)(l_2x + m_2y + n_2)$

$$S^1 = (-l_1x - m_1y + n_1)(-l_2x - m_2y + n_2)$$

Above four lines form a parallelogram

$$\text{Area} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{a_1b_2 - a_2b_1} \right|$$

17. \overline{OP} and \overline{OQ} be the bisectors of the angles between OA and OB

Equations of OB and OA are

$$lx + my = 0 \text{ and } mx - ly = 0$$

$$\text{and } (lx + my)(mx - ly) = 0$$

$$\Rightarrow lmx^2 - (l^2 - m^2)my - lmy^2 = 0$$

Combined equation of OP and OQ is

$$\frac{x^2 - y^2}{2lm} = \frac{xy}{-\frac{1}{2}(l^2 - m^2)}$$

$$\Rightarrow (l^2 - m^2)(x^2 - y^2) + 4lmxy = 0$$

18. $y = |x-1| \Rightarrow y^2 = (x-1)^2$

$$\text{Replace } x \text{ by } -x \Rightarrow y^2 = (-x-1)^2$$

$$\Rightarrow x^2 - y^2 + 2x + 1 = 0$$

19.

$$\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{3}{4x}\right)}{\tan\left(\frac{4}{3x}\right)} = \lim_{x \rightarrow \infty} \frac{\left(\frac{3}{4x}\right)}{\left(\frac{4}{3x}\right)} = \frac{3}{4} \cdot \lim_{x \rightarrow \infty} \left(\frac{3}{4}\right)^x = \frac{3}{4} \cdot (0) = 0$$

20.

$$\lim_{n \rightarrow \infty} \prod_{r=3}^n \left(\frac{r^3 - 8}{r^3 + 8}\right) = \lim_{n \rightarrow \infty} \prod_{r=3}^n \left[\frac{(r-2)(r^2 + 2r + 4)}{(r+2)(r^2 - 2r + 4)}\right]$$

$$\lim_{n \rightarrow \infty} \prod_{r=3}^n \left(\frac{r-2}{r+2}\right) \cdot \prod_{r=3}^n \left(\frac{r^2 + 2r + 4}{r^2 - 2r + 4}\right)$$

$$\lim_{n \rightarrow \infty} \prod_{r=3}^n \left(\frac{r-2}{r+2}\right) \cdot \prod_{r=3}^n \left[\frac{(r+1)^2 + 3}{(r-1)^2 + 3}\right]$$

$$= \lim_{n \rightarrow \infty} \frac{1.2.3.4 \dots (n^2 + 3)(n^2 + 2n + 4)}{(n-1)n(n+1)(n+2) \dots 7.12}$$

$$\lim_{n \rightarrow \infty} \frac{2 \left(1 + \frac{3}{n^2}\right) \left(1 + \frac{2}{n} + \frac{4}{n^2}\right)}{7 \left(1 - \frac{1}{n}\right) \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right)} = \frac{2}{7}$$

NUMERICAL VALUE HINTS & SOLUTIONS

21. We have $\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$

$$= \begin{vmatrix} 0 & -2a & -2a \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$$

$$= -2a \begin{vmatrix} 0 & 1 & 1 \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$$

$$= (-2a)(-bc - bc)$$

22.

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$

$$= 2(x+y+z)(x+y+z)(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 2(x+y+z)^3 [(1)(1-0)] = 2(x+y+z)^3$$

$$= k(x+y+z)^3$$

$$\therefore k = 2$$

23. We have, $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$

$$= \frac{xyz}{xyz} \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = -\Delta \Rightarrow \Delta_1 + \Delta = 0$$

24. $15x^2 + 31xy + 14y^2 = 0$

$$\Rightarrow 15x^2 + 21xy + 10xy + 14y^2 = 0$$

$$\Rightarrow 3x(5x + 7y) + 2y(5x + 7y) = 0$$

$$\Rightarrow (5x + 7y)(3x + 2y) = 0$$

$$\Rightarrow 5x + 7y = 0, \Rightarrow 3x + 2y = 0$$

$$P_1 = \frac{6+6}{\sqrt{9+4}} = \frac{12}{\sqrt{13}}; P_2 = \frac{10+21}{\sqrt{25+49}} = \frac{31}{\sqrt{74}}$$

$$\frac{1}{5} \left(13P_1^2 + \frac{74}{31} P_2^2 \right) = \frac{1}{5} \left(13 \times \frac{144}{13} + \frac{74}{31} \times \frac{31 \times 31}{74} \right)$$

$$= \frac{1}{5} (175) = 35$$

25. $a + b = \sin^2 \theta + \cos^2 \theta - 1 = 0 \Rightarrow \phi = 90$

PHYSICS

26. (2)
27. Potential energy of the spring will transfer to the kinetic energy of air molecules.
28. As $K = \frac{P^2}{2m}; K \propto \frac{1}{m}$.

29. We know that $K = \frac{1}{2}mv^2$

\Rightarrow It represent a parabola about y -axis

30. $W = K_f - K_i$.

$$\text{or } \int_3^4 (-6x) dx = \frac{1}{2} \times 2 (v_f^2 - 8^2)$$

$$\text{or } \left| \frac{-6x^2}{2} \right|_3^4 = v_f^2 - 64$$

$$\text{or } -3|4^2 - 3^2| = v_f^2 - 64$$

$$\therefore v_f = 6.6 \text{ m/s}$$

31. $\frac{1}{2}kx^2 = mgh$

$$\text{or } h = \frac{kx^2}{2mg}$$

32. In the device when free end of the string is pulled by l , the block will rise by $\frac{l}{2}$. So increase in potential energy $U = mgl/2$.

33. The tension in the string, $T = F$.

Also $W_{gravity} + W_{tension} = \Delta K$

$$\therefore W_{tension} = \Delta K - W_{gravity}$$

34. $v = \frac{dx}{dt} = \frac{d(3t - 4t^2 + t^3)}{dt} = 3 - 8t + 3t^2$

$t = 0, 3 \text{ m/s}$

$t = 4, v_f = 3 - 8 \times 4 + 3 \times 4^2 = 19 \text{ m/s}$

Now, $W = \frac{1}{2}m(v_f^2 - v_i^2)$

$$= \frac{1}{2} \times 30 \times 10^{-3} [19^2 - 3^2] = 5.28 \text{ J}$$

35. $W = \frac{1}{2}m(v_f^2 - v_i^2)$

$$= \frac{1}{2} \times 100 \times 10^{-3} (0^2 - 5^2)$$

$$= -1.25 \text{ J}$$

36. $mg(100 - 20) = \frac{1}{2}mv^2$

$$\therefore v = 40 \text{ m/s}$$

37. $\frac{1}{2}kx^2 = 2 \times \frac{1}{2}mv^2$

38. $W = \frac{Mg \times l}{2} = \frac{Mgl}{2}$

39. $Fx = \frac{1}{2}m(v_f^2 - v_i^2)$

or

$$F \times 200 \times 10^{-3} = \frac{1}{2} \times 60 \times 10^{-3} (300^2 - 600^2)$$

- $\therefore F = 40.5 \times 10^3 N$
40. Given, $U = 20xyz^{-1}$
- $$F_x = -\frac{\partial U}{\partial x} = -\frac{20y}{z}$$
- $$F_y = -\frac{\partial U}{\partial y} = -\frac{20x}{z}$$
- and $F_z = -\frac{\partial U}{\partial z} = -\frac{20xy}{z^2}$
- $$\therefore \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$
41. See examples
42. $W_{gravity} + W_F = \Delta K$
- $$-10 \times g(10 - 10 \cos 60^\circ) + 200 \times 10 \sin 60^\circ$$
- $$= \frac{1}{2} \times 10 \times v_f^2 - 0$$
- $$v_f = 17 m/s$$
43. $J = Ft = \Delta P$
44. $\Delta KE = \frac{1}{2} m(v_2^2 - v_1^2), J = \Delta p = m(v - u)$
45. Both the impulse and change in momentum are in same direction.
46. $\vec{F} \cdot \vec{S} = 0$
- or $(6\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} - 3\hat{j} - x\hat{k}) = 0$
- or $6 \times 2 - 2 \times 3 - 3x = 0$
- $$\therefore x = 2$$
47. At $x = 0, v_i = 5(0)^{\frac{3}{2}} = 0$
- and $x = 2m, v_f = 5(2)^{\frac{3}{2}} = 5\sqrt{8} m/s$
- $$W = \Delta K$$
- $$= \frac{1}{2} m(v_f^2 - v_i^2)$$
- $$= \frac{1}{2} \times 0.5 \left[(5\sqrt{8})^2 - 0^2 \right]$$
- $$= 50 J$$
48. $W = \text{area of } \vec{F} - x$
- $$= m \left[\text{area of } \vec{a} - x \right]$$
- $$= 2 \left[\frac{1}{2} (5+3) \times 6 - \frac{1}{2} (2+1) \times 6 \right] = 30 J$$
49. $v_i = (8\hat{i} + 9\hat{j}) m/s$
- and $\vec{v}_f = 8\hat{i} + (9 - g \times 5)\hat{j} = (8\hat{i} - 41\hat{j}) m/s$
- so $v_i = \sqrt{8^2 + 9^2} = \sqrt{147}$
- and $v_f = \sqrt{8^2 + 41^2} = \sqrt{1745}$
- $$\Delta U = -\Delta K = \frac{1}{2} \times 1 \times [145 - 1745]$$

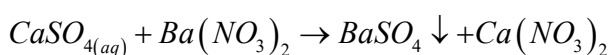
$$= -800 J$$

$$= \left(\frac{-20y}{z} \right) \hat{i} - \left(\frac{20x}{z} \right) \hat{j} + \frac{20xy}{z^2} \hat{k}$$

50. $J = \int F \cdot dt$

CHEMISTRY

51. One g atom = mass of N_A atoms
- $$1.008 g = 6.022 \times 10^{23} \text{ H-atoms}$$
- $$\text{Mass of one H-atom} = \frac{1.008}{6.022 \times 10^{23}} = 1.67 \times 10^{-24} g$$
52. Mass of water escaped = $10.407 - 9.520 = 0.887 g BaI_2 \cdot xH_2O$
- $$BaI_2(9.520g) \quad xH_2O(0.887g)$$
- $$\frac{\text{moles of } H_2O}{\text{moles of } BaI_2} = x$$
- $$= \frac{0.887/18}{9.520/391} = x = x = 2$$
53. Law of multiple proportions
54. $\text{Mole of Glucose} = \frac{6.02 \times 10^{22}}{6.02 \times 10^{23}} = 0.1$
- $$M_{glucose} = \frac{0.1 \times 100}{50} = 2$$
- $$m = \frac{15}{98 \times \frac{(100 \times 1.1 - 15)}{1000}} = 1.6$$
55. g-atoms of $X = \frac{75.8}{75}$ and
- $$\text{g-atom of } Y = \frac{24.2}{16}$$
57. Limiting reagent is one which is completely consumed in reaction
58. $2BCl_3 + 3H_2 \rightarrow 2B + 6HCl$
- $$2 \times 10.8g \text{ Boron} = 3 \times 22.4L H_2$$
- $$21.6g \text{ Boron} = \frac{3 \times 22.4 \times 21.6}{2 \times 10.8}$$
- $$= 67.2L H_2$$
59. Oxidation number remains unchanged.
60. Meq. Of $KMnO_4$
- $$= 4 \times 1000 \times 0.5 = \frac{\omega}{31.6} \times 1000$$
- $$\omega = 6.32 g$$
- 61.



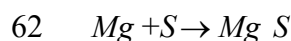
Mass of $CaSO_4$ required to get 0.617

g $BaSO_4$

$$= \frac{136}{233} \times 0.617 = 0.360 \text{ g } CaSO_4$$

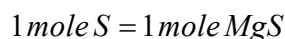
% Purity of $CaSO_4$ sample

$$\frac{0.360}{0.455} \times 100 = 79.15\%$$



Mass ratio = 24 g 32 g

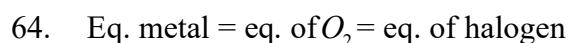
When 2g S (2/32 mole) reacts with Mg. It acts as limiting reagent while Mg is excess reagent. Thus amount produced will be decided by moles of S not by moles of Mg, thus



$$\text{Moles of } MgS \text{ formed} = \frac{2}{32}$$

And mass of

$$MgS \text{ formed} = \frac{2}{32} \times 56 = 3.5 \text{ g}$$



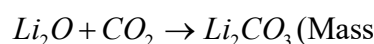
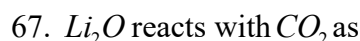
$$\text{Eq. of } O_2 = \frac{0.1}{E} = \frac{0.1}{32} = \frac{0.1}{8}$$

$$\text{Eq. of halogen} = \frac{1}{E} = \frac{0.1}{8}$$

$$E = 80$$



$$66. \frac{1}{1+4} = \frac{1}{5} \therefore x_{H_2O} = 4/5$$



of $Li_2O = 30 \text{ g})$

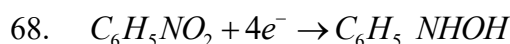
1 mole of Li_2O reacts with 22.4 L CO_2 at

NTP

$$1000 \text{ g } Li_2O \text{ absorbs } \frac{22.4 \times 1000}{30}$$

$$= 746.66 \text{ litre } CO_2$$

\therefore Absorption efficiency is 746.66 L/kg

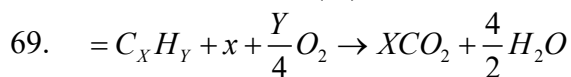


$$4 \times 6 + 5 + x - 4 = 0 \qquad 24 + 7 + x - 2 = 0$$

$$x = -25 \qquad \qquad \qquad x = -29$$

$$n = -25 - (-29) = 4$$

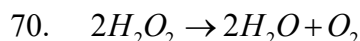
$$1 \text{ mole} = 4F \quad (\text{or}) \quad 0.1 \text{ mole} = 0.4F$$



$$3.08 \text{ g} \qquad 0.72 \text{ g}$$

$$0.07 \text{ moles } 0.04 \text{ moles}$$

$$\frac{X}{Y/2} = \frac{0.07}{0.04} = \frac{X}{Y} = \frac{7}{8}$$



$$68 \text{ g} \qquad 22.4 \text{ l (at STP)}$$

Mass of H_2O_2 in 1.5 N solution =

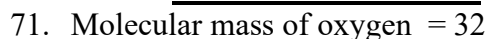
$$\text{E. wt. of } H_2O_2 \times 1.5N$$

$$= 17 \times 1.5 = 25.5 \text{ g}$$

= Volume strength of 1.5N H_2O_2 solution

$$\frac{22.4 \text{ l} \times 25.5 \text{ g}}{68.0} = 8.4 \text{ l}$$

NUMERICAL HINTS



$$\text{V.D of oxygen} = \frac{32}{2} = 16$$

$$\text{V.D of gaseous element } 16 \times 5 = 80$$

Molecular mass of gaseous element = $80 \times 2 = 160$

As molecule is Triatomic, Its atomic mass

$$= \frac{\text{molar mass}}{\text{Atomicity}}$$

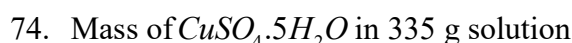
$$= \frac{160}{3} = 53.33$$

Ans: 53.33

72. $\frac{75}{100} \times V \times 0.8 = \frac{30}{100} \times 150 \times 0.9$

$$V = 67.5 \text{ mL}$$

73. $Fe = \frac{5}{100} \times 54 + \frac{90}{100} \times 56 + \frac{5}{100} \times 57$
 $= 55.95 \text{ amu}$



$$\frac{32 \times 335}{100} = 107.20 \text{ g}$$

$$\text{Mass of water} = 335 - 107.20 = 227.80 \text{ g}$$

At 0°C mass of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ in 100 g of solution = 12.5 g

Mass of water = $100 - 12.5 \text{ g} = 87.5 \text{ g}$

$87.5 \text{ H}_2\text{O} \rightarrow 12.5$

$227.80 \rightarrow ?$

$$\frac{12.5 \times 227.8}{87.5} = 32.73 \text{ g}$$

Amount of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ crystallised

$$= 107.20 - 32.73 = 74.47$$

$$75. \text{ Wt. of } \text{KMnO}_4 = 4 \times 1000 \times 0.5 = \frac{w}{31.6} \times 1000$$

$$w = 6.32 \text{ g}$$