

# MELUHA INTERNATIONAL SCHOOL

INDIA

SR MPC

JEE MAINS MODEL

29-03-2020

## SECTION – I

### (SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.**

## MATHS

### SYLLABUS: Matrices (inverse and linear equations), Differentiation

1. If  $A+B+C = \pi$ , then the value of the determinant  $D = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$  is:
- A) 1                                      B) -1                                      C) 0                                      D) None of these
2. A divisor of the determinant  $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$  is:
- A)  $1+a^2+b^2+c^2$                       B)  $a^2+b^2+c^2$                       C)  $(a+b+c)^2$                       D)  $ab+bc+ca$
3. If  $\alpha, \beta$  and  $\gamma$  are the roots of the equation  $x^3 + px + q = 0$ , then the value of the determinant  $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$
- A)  $p$                                       B)  $q$                                       C)  $p^2 - 2q$                                       D) None of these
4. The determinant  $\begin{vmatrix} \cos(\theta - \phi) & -\sin(\theta + \phi) & \cos 2\phi \\ \sin \theta & \cos \theta & \sin \phi \\ -\cos \theta & \sin \theta & \cos \phi \end{vmatrix}$  is:
- A) 0                                      B) Independent of  $\theta$   
C) Independent of  $\phi$                       D) Independent of  $\theta$  and  $\phi$  both
5. The determinant  $\begin{vmatrix} \cos(x-y) & \cos(y-z) & \cos(z-x) \\ \cos(x+y) & \cos(y+z) & \cos(z+x) \\ \sin(x+y) & \sin(y+z) & \sin(z+x) \end{vmatrix}$  is:
- A)  $2 \sin(x-y) \sin(y-z) \sin(z-x)$                       B)  $-2 \sin(x-y) \sin(y-z) \sin(z-x)$   
C)  $2 \cos(x-y) \cos(y-z) \cos(z-x)$                       D)  $-2 \cos(x-y) \cos(y-z) \cos(z-x)$
6. The determinant  $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$  is:
- A)  $2abc$                                       B)  $3abc$                                       C)  $4abc$                                       D) None of these

7. The set of homogeneous equations  $tx + (t+1)y + (t-1)z = 0$ ,  $(t+1)x + ty + (t+2)z = 0$ ,  $(t-1)x + (t+2)y + tz = 0$  has non-trivial solution for:
- A) Three values of  $t$     B) Two values of  $t$     C) One value of  $t$     D) No value of  $t$
8. If  $a, b, c$  are real then the value of determinant  $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1$  if:
- A)  $a+b+c=0$     B)  $a+b+c=1$     C)  $a+b+c=-1$     D)  $a=b=c=0$
9. The equation  $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$
- A) Has no real solution  
 B) Has 4 real solutions  
 C) Has two real and two non-real solutions  
 D) Has infinite number of solutions, real or non-real
10. The solution of the following equations:  
 $x - (4k-3)y + 2z = 0$ ,  $kx - (2k-1)y + (k+1)z = 0$ ,  $(2k+2)x + 3ky + (k+2)z = 0$  For  $k = 1$ ,  $(x, y, z)$  is given by:
- A)  $(t, 3t, t)$     B)  $(-2t, -10t, -4t)$     C)  $(9t, -5t, -7t)$     D)  $(4t, 6t, t)$
11. If  $(\sin x)(\cos y) = \frac{1}{2}$  then  $\frac{d^2y}{dx^2}$  at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$  is
- A)  $-4$     B)  $-2$     C)  $-6$     D)  $0$
12. If  $t(1+x^2) = x$  and  $x^2 + t^2 = y$  then at  $x = 2, \frac{dy}{dx} =$
- A)  $\frac{24}{5}$     B)  $\frac{101}{125}$     C)  $\frac{488}{125}$     D)  $\frac{358}{125}$
13. If  $f(x) = |x-2|$  and  $g(x) = f[f(x)]$  then  $\sum_{r=0}^3 g'(2r-1)$  is equal to
- A)  $0$     B)  $1$     C)  $-1$     D)  $2$
14. If  $y = \frac{1}{x}$  then  $\frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} =$
- A)  $0$     B)  $1$     C)  $x/y$     D)  $y/x$
15. If  $x = \frac{1+t}{t^3}$  and  $y = \frac{3+4t}{2t^2}$  then  $x(y')^3 =$
- A)  $1-y'$     B)  $1+y$     C)  $y'-1$     D)  $y'$
16. The value of  $y''(1)$ , if  $x^3 - 2x^2y^2 + 5x + y - 5 = 0$  when  $y(1) = 1$  is equal to
- A)  $\frac{22}{7}$     B)  $-7\frac{21}{28}$     C)  $8$     D)  $-8\frac{22}{27}$
17. If  $f'(3) = 2$  then  $\lim_{h \rightarrow \infty} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$  is
- A)  $1$     B)  $2$     C)  $3$     D)  $1/2$
18. If  $f(x) = \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$  then  $f'(x)$  is equal to
- A)  $\sec x(\tan x - \sec x)$     B)  $\sec x(\sec x - \tan x)$     C)  $\sec x(\sec x + \tan x)$     D) None of these

19. If  $y = \tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$ , then  $\frac{dy}{dx}$  is equal to  
 A)  $2\sec^2(x/2)$       B)  $(1/2)\sec^2(x/2)$       C)  $(1/2)$       D)  $-(1/2)\sec^2(x/2)$
20.  $f(x) = \begin{cases} |x-4| & \text{for } x \geq 1 \\ \frac{x^3}{2} - x^2 + 3x + 1/2 & \text{for } x < 1 \end{cases}$  then  
 A)  $f(x)$  is continuous at  $x=1$  and  $x=4$       B)  $f(x)$  is differentiable at  $x=4$   
 C)  $f(x)$  is continuous and differentiable at  $x=1$       D)  $f(x)$  is only continuous at  $x=1$

**SECTION-II**

**(Numerical Value Answer Type)**

21. Let  $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$  and suppose that  $\det(A) = 2$  then the  $\det(B)$  equals, where  
 $B = \begin{bmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{bmatrix}$
22. Let  $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$ , then the maximum value of  $f(x)$  is:
23. Let  $f(\theta) = \begin{vmatrix} \cos^2 \theta & \cos \theta \sin \theta & -\sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta & \cos \theta \\ \sin \theta & -\cos \theta & 0 \end{vmatrix}$  then  $f\left(\frac{\pi}{6}\right)$  is:
24. If  $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ ,  $-\pi < x < \pi$  then  $\frac{dy}{dx} =$
25. Let  $y = x^3 - 8x + 7$  and  $x = f(t)$ . If  $\frac{dy}{dt} = 2$  and  $x = 3$  at  $t = 0$  then the value of  $\frac{dx}{dt}$  at  $t = 0$  \_\_\_\_\_

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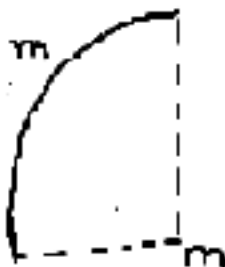
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**PHYSICS**

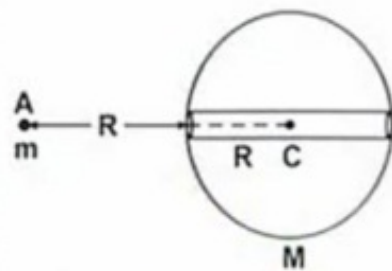
**SYLLABUS: Rotational Motion, Gravitation**

26. Moment of inertia of solid sphere about its diameter is  $I$ . If that sphere is recast into 8 identical small spheres, then the moment of inertia of small sphere about its diameter is:  
 A)  $\frac{I}{8}$       B)  $\frac{I}{16}$       C)  $\frac{I}{24}$       D)  $\frac{I}{32}$
27. Three particles each of mass ‘ $m$ ’ are arranged at the corner of an equilateral triangle of side ‘ $L$ ’. If one of the masses is doubled, the shift in the centre of mass of the system  
 A)  $\frac{L}{\sqrt{3}}$       B)  $\frac{L}{4\sqrt{3}}$       C)  $\frac{\sqrt{3}L}{4}$       D)  $\frac{L}{2\sqrt{3}}$

28. A uniform thin bar of mass  $6m$  and length  $12L$  is bent to make a regular hexagon. Its moment of inertia about an axis passing through centre of mass and perpendicular to the plane of the hexagon is:
- A)  $20 mL^2$                       B)  $6 mL^2$                       C)  $\frac{12}{5} mL^2$                       D)  $30 mL^2$
29. A thin rod of length  $L$  and mass  $M$  is held vertically with one end on the floor and is allowed to fall. Find the velocity of the other end when it hits the floor, assuming that the end on the floor does not slip
- A)  $\sqrt{\frac{3g}{L}}$                       B)  $\sqrt{3gL}$                       C)  $\sqrt{\frac{L}{3g}}$                       D)  $\sqrt{\frac{g}{3L}}$
30. A cylinder of mass  $M$ , radius  $R$  is resting on a horizontal platform (which is parallel to  $XY$  plane) with its axis fixed along the  $Y$  axis and free to rotate about its axis. The platform is given a motion in  $X$  - direction given by  $x = A \cos \omega t$ . There is no slipping between cylinder and platform. The maximum torque acting on the cylinder during its motion is:
- A)  $\frac{1}{2} MRA\omega^2$                       B)  $MRA\omega^2$                       C)  $2MRA\omega^2$                       D)  $2MRA\omega^2 \cos \omega t$
31. Find the gravitational attraction on mass  $m$  due to the quarter ring of same mass  $m$  and Radius ' $R$ ' as shown in figure.



- A)  $\frac{2\sqrt{2}Gm^2}{\pi R^2}$                       B)  $\frac{2Gm^2}{\pi R^2}$                       C)  $\frac{Gm^2}{\pi R^2}$                       D)  $\frac{\sqrt{2}Gm^2}{\pi R^2}$
32. If diameter of a planet is four times that of earth. What is the time period of a pendulum on that planet, if it is a second pendulum on earth. Take mean density of the planet equal to that of earth.
- A) 4 sec                      B) 1 sec                      C) 2 sec                      D) 3 sec
33. Two particles of masses  $m_1$  and  $m_2$  are initially at rest at infinite separation. When released, what is their relative velocity of approach when they are at a separation  $d$ .
- A)  $\sqrt{\frac{G(m_1 + m_2)}{2d}}$                       B)  $\sqrt{\frac{G(m_1 + m_2)}{d}}$                       C)  $\sqrt{\frac{2G(m_1 + m_2)}{d}}$                       D)  $\sqrt[2]{\frac{2G(m_1 + m_2)}{d}}$
34. Figure shows a fixed solid sphere of mass  $M$  and radius  $R$  with a narrow smooth tunnel along its diameter. Another particle of mass  $m$  is placed at point A as shown. If the particle is released from rest, then its speed when it reaches the center of sphere is



- A)  $\sqrt{\frac{GM}{R}}$                       B)  $\sqrt{\frac{2GM}{R}}$                       C)  $\sqrt{\frac{GM}{2R}}$                       D)  $2\sqrt{\frac{GM}{R}}$

35. Two satellites A and B of same mass are orbiting earth at altitudes R and 3R respectively, where R is Radius. The earth taking their orbits to be circular, find the ratio of their kinetic energies.  
 A) 1 : 3                      B) 1 : 1                      C) 2 : 1                      D) 3 : 1
36. A Satellite of mass  $2 \times 10^3$  kg is to be shifted from on orbit of radius 2R to another orbit of radius 3R. Calculate the minimum energy required for this. [R is the Radius of earth]  
 [  $g = 10 \text{ms}^{-2}$  ]  
 A)  $1.066 \times 10^{10} \text{J}$               B)  $1.066 \times 10^9 \text{J}$               C)  $2.132 \times 10^{10} \text{J}$               D)  $2.132 \times 10^9 \text{J}$
37. An artificial satellite is moving in a circular orbit around earth with a speed equal to half the escape Velocity from earth surface. Find the height of satellite above the earth surface.  
 A) 3200 Km                      B) 6400 Km                      C) 10800 Km                      D) 19600 Km
38. The radius of a planet is  $R_1$  and a satellite revolves around it in a circular orbit of radius  $R_2$ . The period of revolution is T. What is the acceleration due to gravity on the surface of the planet?  
 A)  $\frac{4\pi^2 R_1^3}{R_2^2 T^2}$                       B)  $\frac{\pi^2 R_1^3}{R_1 T}$                       C)  $\frac{4\pi^2 R_2^3}{R_1^2 T^2}$                       D)  $\frac{\pi^2 R_1^3}{R_2^2 T^2}$
39. A satellite revolves around a planet in an elliptical orbit. Its perigee and apogee are  $0.5 \times 10^7 \text{ m}$  and  $1.5 \times 10^7 \text{ m}$  respectively. If the minimum speed of satellite in orbit is  $5 \times 10^3 \text{ m/s}$ , then its maximum speed in orbit is  
 A)  $15 \times 10^4 \text{ m/s}$               B)  $\frac{5}{3} \times 10^3 \text{ m/s}$               C)  $1.5 \times 10^4 \text{ m/s}$               D)  $\frac{5}{3} \times 10^4 \text{ m/s}$
40. The change in the value of 'g' at a height 'h' above the surface of earth is same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?  
 A)  $d = \frac{h}{2}$                       B)  $d = \frac{3h}{2}$                       C)  $d = 2h$                       D)  $d = h$
41. The distance of the centres of moon and earth is d. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational field will be zero.  
 A)  $\frac{d}{2}$                       B)  $\frac{2d}{3}$                       C)  $\frac{4d}{3}$                       D)  $\frac{9d}{10}$
42. A point P lies on the axis of a ring of mass M and radius R at a distance R from its centre. A small particle starts from p and reaches the centre under gravitational attraction only. It's speed at center will be  
 A)  $\sqrt{\frac{2GM}{R}}$                       B)  $\sqrt{\frac{2GM}{R} \left[ 1 - \frac{1}{\sqrt{2}} \right]}$                       C) Zero                      D)  $\sqrt{\frac{GM}{R}}$
43. The gravitational field in a region is given by  $\vec{E} = (4i + j) \text{ N/Kg}$ . Work done by this field is Zero when a particle is moved along the line is  
 A)  $y + 4x = 2$                       B)  $4y + x = 6$                       C)  $x + y = 5$                       D) All of these
44. Two masses 90 Kg and 160 Kg are at a distance 5 m apart. Find the magnitude of intensity of the gravitational field at a point which is at a distance 3 m from 90 Kg and 4 m from 160 Kg mass.  
 A)  $10\sqrt{2} \text{ G N Kg}^{-1}$               B)  $10 \text{ G N Kg}^{-1}$               C)  $7 \text{ G N Kg}^{-1}$               D)  $2 \text{ G N Kg}^{-1}$
45. Find the height from the earth's surface where acceleration due to gravity is 25% of it's value on the surface of earth. [R=6400Km]  
 A) 3200 Km                      B) 6400 Km                      C) 10800 Km                      D) 19600 Km

**SECTION- II**

**(Numerical Value Answer Type)**

46. Two kids weighing 10 kg and 15 kg respectively, are trying to balance a see saw of total length 5.00 m, with its fulcrum at the centre. If one of the kids is sitting at an end, then what is the distance in meter from the fulcrum to the other kid \_\_\_\_\_
47. A small hole is made in a disc of mass M and radius R at a distance  $\frac{R}{4}$  from center. The disc is supported on a horizontal peg through this hole. The moment of inertia of the disc about the peg is  $\frac{9}{a}$ . The value of a is \_\_\_\_\_
48. A force of 490 N acts tangentially at the highest point of a sphere of mass 5 kg kept on a horizontal plane. If the sphere rolls without slipping, then the acceleration of center of sphere in  $m/s^2$  is \_\_\_\_\_
49. A solid sphere is projected up along an inclined plane of inclination  $\theta = 30^\circ$  with a speed  $V = 2ms^{-1}$ . If it rolls without slipping, then the maximum distance traversed by it in m is ( $g = 10ms^{-2}$ )
50. A child is standing with folded hands at the center of a platform rotating about its central axis. The K.E of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The K.E of the system is now BK. The value of B is \_\_\_\_\_

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**CHEMISTRY**

**SYLLABUS: I-A, II-A, III-A & IV-A Groups, Junior organic Chemistry (GOC) and Alkanes**

51. Find out 'Z' in the following sequence of reaction  $Na \xrightarrow{\text{Air}} x \xrightarrow{\text{Moisture in air}} y \xrightarrow{\text{Cu}} z$   
 1)  $Na_2CO_3$                       2)  $NaOH$                       3)  $Na_2O_2$                       4)  $Na_2O$
52. Aluminum reacts with  $NaOH$  and forms compound 'X' If the co-ordination number of Al in 'X' is 6, the correct formula, of X is  
 1)  $[Al(H_2O)_4(OH)_2]^+$     2)  $[Al(H_2O)_3(OH)_3]$     3)  $[Al(H_2O)_2(OH)_4]^-$     4)  $[Al(H_2O)_6](OH)_3$
53. Amongst  $LiCl, RbCl, BeCl_2$  and  $MgCl_2$  the compounds with the greatest and the least ionic character, respectively are :  
 1)  $LiCl$  and  $RbCl$     2)  $RbCl$  and  $BeCl_2$   
 3)  $MgCl_2$  and  $BeCl_2$     4)  $RbCl$  and  $MgCl_2$
54. Magnesium burns in air giving two products. Their formulae are  
 1)  $MgO, MgN$                       2)  $MgO_2, Mg_3N_2$                       3)  $MgO, Mg_3N_2$                       4)  $Mg_2O, Mg_3N_2$
55. A piece of magnesium is burnt in air to give ash. Then ash is dissolved in water to liberate a colorless gas, which turns red litmus to blue. The colorless gas is  
 1)  $NH_3$                       2)  $NO_2$                       3)  $N_2$                       4)  $NO$
56. Sedimentary rocks laid down under water mainly contain  
 1)  $CaO$                       2)  $Ca(OH)_2$                       3)  $CaCO_3$                       4)  $CaSO_4$

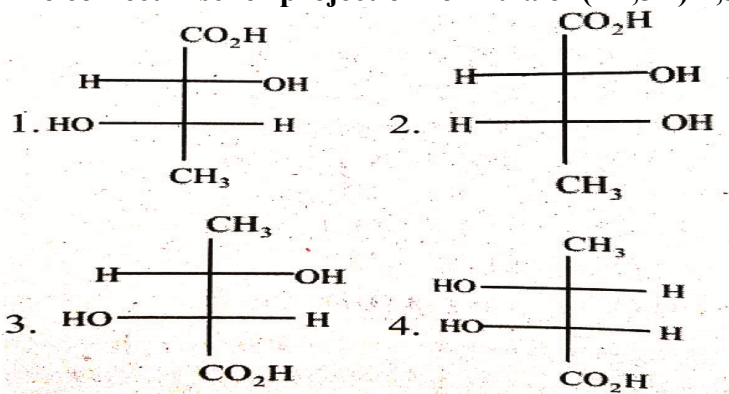
57. The property not associated with refractory nature of Mg is  
 1) it has high m.p  
 2) it is a good conductor of heat  
 3) it is chemically inert as well as electrical insulator  
 4) forms Grignard's reagent
58.  $BCl_3 + LiAlH_4 \rightarrow A + LiCl + AlCl_3$   
 $A + H_2O \rightarrow B + H_2$ ,  $B \xrightarrow{\text{Red heat}} C$ . In this reaction sequence A,B and C compounds respectively are  
 1)  $B_2H_6, B_2O_2, B$       2)  $B_2H_6, H_3BO_3, B_2O_3$       3)  $B_2H_6, H_3BO_3, B$       4)  $HF, H_3BO_3, B_2O_3$
59. Which amongst the following is also called as a sesqui oxide  
 1)  $B_2O_3$       2)  $Al_2O_3$       3)  $Tl_2O_3$       4) all
60. Which of the following is a lewis acid  
 1)  $AlCl_3$       2)  $MgCl_2$       3)  $CaCl_2$       4)  $BaCl_2$
61. Silica reacts with hydride of super halogen to form 'X' on hydrolysis of X another compound Y on heating to form Z. The 'Z' can also be prepared in the following reaction.  
 1)  $SiO_2 + 4HF \rightarrow SiF_4 + 2H_2O$       2)  $Si + O_2 \rightarrow SiO_2$   
 3)  $K_2SiF_6 + 4K \rightarrow 6KF + Si$       4)  $Si + 2NaOH + H_2O \rightarrow Na_2SiO_3 + 2H_2O$
62. Which glass has the highest percentage of lead?  
 1) Soda glass      2) Flint glass      3) Jena glass      4) Pyrex glass
63. Addition of sodium hydroxide solution to a weak acid (HA) results in a buffer of  $P^H$  is 6. If ionisation constant of HA is  $10^{-5}$ , the ratio of salt to acid concentration in the buffer solution will be  
 1) 5:4      2) 1:10      3) 4:5      4) 10:1
64. Decreasing order of 'P' Character in the following  
 A :  $SiO_2$ ; B :  $CO_2$ ; C : Graphite  
 1)  $A > B > C$       2)  $B > A > C$       3)  $B > C > A$       4)  $A > C > B$
65. The difference in properties of  $CH_4$  and  $SiH_4$  is due to  
 1) Large difference in the electronegativity of carbon and silicon  
 2) Less difference in the size of carbon and Silicon atoms  
 3) The inability of carbon to expand its octet  
 4) The inability of silicon to form double bonds
66. The IUPAC name of the alkane is



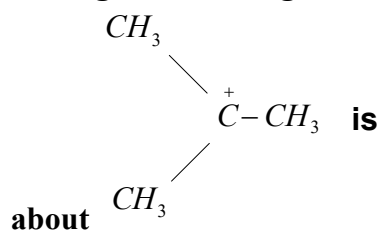
is

- 1) 2- Isopropyl-2, 6, 6-trimethylheptane  
 2) 5 tert-butyl-2isopropyl-2-methylpentane  
 3) 2, 3, 3, 7, 7-Pentamethyloctane  
 4) 2, 2, 6, 6, 7-Pentamethyloctane

67. The correct Fischer projection formula of (2R,3R)-2,3-dihydroxy butanoic acid is

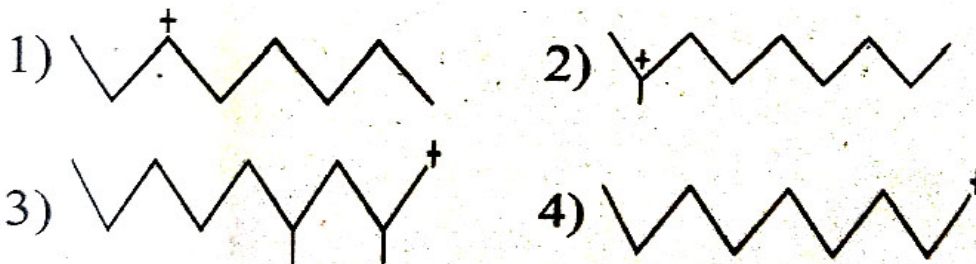


68. Among the following, the true property



- 1) non-planar
- 2)  $C^+$  is  $sp^2$ -hybridized
- 3) electrophile can attack  $C^+$
- 4) does not undergo hydrolysis

69. Select the most stable carbocation from amongst the following



70. Polarization of electrons in acrolein may be written as

- 1)  $CH_2^{\delta-} = CH - C^{\delta+} = O$
- 2)  $CH_2^{\delta-} = CH - CH = O^{\delta+}$
- 3)  $CH_2^{\delta-} = CH^{\delta-} - CH = O$
- 4)  $CH_2^{\delta-} = CH - CH = O^{\delta+}$

**NUMERICAL VALUE QUESTIONS**

- 71. How many of the following elements of I A group are lighter than water (i) Cs (ii) Na (iii) K (iv) Li
- 72. How many of the following will not give any colour in flame (i) Be (ii) Mg (iii) Na (iv) Li
- 73. Magnesium oxide when mixed with a saturated solution of magnesium chloride sets to a hard mass like cement known as "sorel cement". The composition of Sorel cement is  $MgO \cdot nMgCl_2 \cdot xH_2O$ . What is the value of n
- 74. On heating calcium ammoniate, ammonia and hydrogen are evolved. How many moles of ammonia are evolved when 1.5 moles of calcium ammoniate are heated
- 75. The ratio of copper and tin in bell metal alloy is \_\_\_\_\_.



**MELUHA INTERNATIONAL SCHOOL**

INDIA

SR MPC

**JEE MAINS MODEL**

29-03-2020

**KEY SHEET****MATHS**

1) 3	2) 1	3) 4	4) 2	5) 2	6) 3	7) 3	8) 4	9) 4	10) 3
11) 1	12) 3	13) 1	14) 1	15) 2	16) 4	17) 1	18) 3	19) 2	20) 1
21) -16	22) 6	23) 1	24) 0.5	25) 0.11					

**PHYSICS**

26) 4	27) 2	28) 1	29) 2	30) 1	31) 1	32) 2	33) 3	34) 2	35) 3
36) 1	37) 2	38) 3	39) 3	40) 3	41) 4	42) 2	43) 1	44) 1	45) 2
46) 1.67	47) 16	48) 140	49) 0.56	50) 0.5					

**CHEMISTRY**

51) 1	52) 3	53) 2	54) 3	55) 1	56) 3	57) 4	58) 2	59) 4	60) 1
61) 2	62) 2	63) 4	64) 4	65) 1	66) 4	67) 2	68) 2	69) 2	70) 4
71) 3	72) 2	73) 5	74) 8	75) 4					

## MELUHA INTERNATIONAL SCHOOL

INDIA

SR MPC

JEE MAINS MODEL

29-03-2020

HINTS & SOLUTIONSMATHS

- Use:  $R_3 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$
- Multiply  $R_1$  by  $a$ ,  $R_2$  by  $b$  and  $R_3$  by  $c$  and divide the determinant by  $abc$ . Now take  $a, b$  and  $c$

common from  $c_1, c_2$  and  $c_3$ . Now use  $C_1 \rightarrow C_1 + C_2 + C_3$  to get  $(a^2 + b^2 + c^2 + 1) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix}$

Now use:  $C_1 \rightarrow C_1 - C_2$  and  $C_2 \rightarrow C_2 - C_3$  to get the value as 1.

- $D = \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$   
Suppose given  $\alpha + \beta + \gamma = 0$   
As coefficient of  $x^2 = 0$

$$\text{Now, } \det(\alpha + \beta + \gamma) \begin{vmatrix} 1 & 1 & 1 \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

- Directly open by  $R_1$  to get  $\cos^2(\theta + \phi) + \sin^2(\theta + \phi) + \cos 2\phi = 1 + \cos 2\phi$ . Which is independent of  $\theta$
- Expand the determinant using first row and use  $x - y = A$ ,  $y - z = B$  and  $z - x = C$   
 $\Rightarrow A + B + C = 0$
- Put  $a = b = c = 1$ .
- D simplifies to  $-4(2t + 1) = 0$
- Multiply  $R_1$  by  $a$ ,  $R_2$  by  $2$  and  $R_3$  by  $c$  and divide the determinant by  $abc$ . Now take  $a, b$  and  $c$  common from  $c_1, c_2$  and  $c_3$ . Now use  $C_1 \rightarrow C_1 + C_2 + C_3$  to get:

$$(a^2 + b^2 + c^2 + 1) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + 1 & b^2 \\ c^2 & c^2 & c^2 + 1 \end{vmatrix} = 1$$

Now use  $C_1 \rightarrow C_1 - C_2$  and  $C_2 \rightarrow C_2 - C_3$

We get  $1 + a^2 + b^2 + c^2 = 1$

$$\Rightarrow a = b = c = 0$$

- 1<sup>st</sup> two columns of 1<sup>st</sup> determinant are same as 1<sup>st</sup> two rows of 2<sup>nd</sup>. Hence transpose the 2<sup>nd</sup>. Add the two determinants and use  $C_1 \rightarrow C_1 + C_3 \Rightarrow D = 0$
- Put  $k = 1$  and solve
- $\sin x \cdot \cos y = \frac{1}{2}$

$$\Rightarrow \cos x \cdot \cos y - \sin y \cdot \sin x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \cot x \cdot \cot y$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x \cdot \cot y - \operatorname{cosec}^2 y \cdot \cot x \cdot \frac{dy}{dx}$$

$$\text{Now } \left( \frac{dy}{dx} \right)_{\left( \frac{\pi}{4}, \frac{\pi}{4} \right)} = 1$$

$$\Rightarrow \left( \frac{d^2y}{dx^2} \right)_{\left( \frac{\pi}{4}, \frac{\pi}{4} \right)} = -2(1) - 2(1)(1) = -4$$

$$12. \text{ Given } t = \frac{x}{1+x^2} \Rightarrow \frac{dt}{dx} = \frac{1-x^2}{(1+x^2)^2}$$

$$x^2 + t^2 = y \Rightarrow \frac{dy}{dx} = 2x \left[ \frac{(1+x^2)^3 + 1 - x^2}{1-x^4} \right]$$

$$\therefore \frac{dy}{dx} = \frac{2x \left[ (1+x^2)^3 + 1 - x^2 \right]}{(1+x^2)^3}$$

$$13. \quad g(x) = \begin{cases} |x|, & x \leq 2 \\ 4-x, & 2 < x \leq 4 \\ x-4, & 4 < x \end{cases}$$

$$g'(-1) + g'(1) + g'(3) + g'(5) = -1 + 1 - 1 + 1 = 0$$

$$14. \quad \frac{dy}{dx} = -1/x^2$$

$$\frac{dy}{dx} + \frac{\sqrt{1+y^4}}{\sqrt{1+x^4}} = \frac{dy}{dx} + \frac{1}{x^2} = -\frac{1}{x^2} + \frac{1}{x^2} = 0$$

$$15. \quad \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$$

$$16. \text{ Differentiating the given expression, we get } 3x^2 - 4xy^2 - 4x^2y \cdot y' + 5 + y' = 0$$

$$\text{at } x=1, \text{ we have } 3 - 4 \cdot 1 \cdot 1 - 4 \cdot 1 \cdot 1 \cdot y' + 5 + y'(1) = 0$$

$$4 - 3y'(1) = 0 \Rightarrow y'(1) = 4/3$$

$$\text{Differentiating again, we have } 6x - 4y^2 - 8xy \cdot y' - 8xy \cdot y' - 4x^2(y')^2 - 4x^2y \cdot y'' + y'' = 0$$

$$\text{Putting } x=1, y=1 \text{ and } y'(1) = 4/3, \text{ we get } 6 - 4 - 8\left(\frac{4}{3}\right) - 8\left(\frac{4}{3}\right) - 4\left(\frac{16}{9}\right) - 3y''(1) = 0$$

$$y''(1) = -8 \cdot \frac{22}{7}$$

$$17. \quad \lim_{y \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$$

$$= \frac{1}{2} \lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3)}{h^2}$$

$$= \frac{1}{2} f'(3) = 2/2 = 1$$

$$18. \quad f(x) = \frac{\tan^2 x + \sec^2 x + 2 \tan \sec x - 1}{(1 + \tan x)^2 - \sec^2 x}$$

$$= \frac{\tan^2 x + \sec^2 x + 2 \tan x \sec x - 1}{2 \tan x} = \tan x + \sec x$$

Thus  $f'(x) = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x)$

19.  $y = \tan^{-1} \sqrt{\frac{2 \sin^2 x/2}{2 \cos^2 x/2}} = \tan^{-1} \tan(x/2) = x/2$

Hence  $\frac{dy}{dx} = \frac{1}{2}$

20. Since  $g(x) = |x|$  is continuous function and  $\lim_{h \rightarrow 1+} f(x) = 3 \lim_{h \rightarrow 1-} f(x)$  so far  $f$  is a continuous function.

In a particular  $f$  is continuous at  $x = 1$  and  $x = 4$ .  $f$  is clearly not differentiable at  $x = 4$ . Since

$g(x) = |x|$  is not differentiable at  $x = 0$ . Now

$$f'(1+) = \lim_{h \rightarrow 0+} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0+} \frac{|-3+h| - 3}{h} = -1$$

$$f'(1-) = \lim_{h \rightarrow 0+} \frac{(1/2)(1+h)^3 - (1+h)^2 + 3(1+h) + 1/2 - 3}{h}$$

$$f'(1-) = \lim_{h \rightarrow 0-} \frac{(1/2)(1+h)^3 - (1+h)^2 - (h^2 + 2h) + 3h}{h} = 5/2$$

Hence  $f$  is not differentiable at  $x = 1$

21.  $\det(B) = \begin{vmatrix} 4x & 2a & -p \\ 4y & 2b & -q \\ 4z & 2c & -r \end{vmatrix} = (4)(2)(-1) \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

$$= -8 \begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = -8 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$$= -8 \times 2 = -16$$

22. Use:  $R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$  and expand to get  $f(x) = 2 + 4 \sin 2x$ .

23. Put  $\theta = \frac{\pi}{6}$  and find determinant.

24.  $y = \tan^{-1} \left( \frac{1 - \cos x}{\sin x} \right)$

$$y = \tan^{-1} \left( \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right)$$

$$y = \tan^{-1} \left( \tan \frac{x}{2} \right)$$

$$y = \frac{x}{2} \quad \left( \because -\pi < x < \pi \text{ or } \frac{-\pi}{2} < \frac{x}{2} < \frac{\pi}{2} \right)$$

25.  $y = x^3 - 8x + 7$

$$\frac{dy}{dx} = 3x^2 - 8$$

$$t = 0 \Rightarrow x = 3$$

$$\frac{dy}{dx} = 3(3)^2 - 8 = 19$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{19} = 0.1053$$

**PHYSICS**

26.  $I = MR^2$

$$M \propto R^3$$

$$R \propto M^{1/3}$$

$$\frac{R_2}{R_1} = \left( \frac{M_2}{M_1} \right)^{1/3}$$

$$\frac{I_2}{I_1} = \frac{M_2}{M_1} \left( \frac{R_2}{R_1} \right)^2$$

$$\frac{I_2}{I_1} = \frac{M_2}{M_1} \left( \frac{M_2}{M_1} \right)^{2/3}$$

$$I_2 = \frac{I_1}{32}$$

27.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

And also  $y_{cm}$

If one of masses doubled find  $x'_{cm}, y'_{cm}$

$$r = \sqrt{(x_{cm} - x'_{cm})^2 + (y_{cm} - y'_{cm})^2}$$

28.  $I = 6 I_{oneside}$

$$= 6 \left[ \frac{m(2L)^2}{12} + mr^2 \right]$$

Where  $r = \sqrt{3}l$

$$I = 20mL^2$$

29.  $Mg \frac{L}{2} = \frac{1}{2} I \omega^2$

$$Mg \frac{L}{2} = \frac{1}{2} \frac{ML^2}{3} \omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

$$V = r\omega$$

$$V = \sqrt{3gL}$$

30.  $x = A \cos \omega t$

$$V = \frac{dx}{dt} = -A\omega \sin \omega t$$

$$a = -A\omega^2 \cos \omega t$$

$$a_{\max} = A\omega^2$$

$$T_{\max} = I \alpha_{\max}$$

$$= I \frac{a_{\max}}{R}$$

$$= \frac{1}{2} MR^2 \times \frac{A\omega^2}{R}$$

$$T_{\max} = \frac{1}{2} MAR\omega^2$$

31. Mass of element of ring

$$dm = \frac{2m}{\pi} d\theta$$

Force on 'm' due to dm

$$dF = \frac{Gmdm}{R^2}$$

$$dF = \frac{2Gm^2 d\theta}{\pi R^2}$$

Net force on m will be

$$F = \int_{-\frac{\pi}{4}}^{+\frac{\pi}{4}} dF \cos \theta$$

$$F = \frac{2\sqrt{2}Gm^2}{\pi R^2}$$

$$32. \quad g = \frac{GM}{R^2} = \frac{G\rho \frac{4}{3}\pi R^3}{R^2}$$

$$g = G\rho \frac{4}{3}\pi R$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$g \propto R \quad \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{R_2}{R_1}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{4R_1}{R_1}}$$

$$T_2 = 1 \text{ second}$$

33. At separation 'd'

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{Gm_1 m_2}{d}$$

From law of conservation of linear momentum

$$m_1 v_1 = m_2 v_2$$

From (1) and (2)

$$V_1 = \sqrt{\frac{2GM_2^2}{d(m_1 + m_2)}} \quad \text{and}$$

$$V_2 = \sqrt{\frac{2GM_1^2}{d(m_1 + m_2)}}$$

Relative velocity

$$V_{rel} = V_1 + V_2$$

34. potential due to sphere at point A,  $V_A = \frac{-GM}{2R}$

Potential due to sphere at point C,  $V_C = \frac{-3}{2} \frac{GM}{R}$

Work done  $W = +[V_A - V_C] \rightarrow (1)$

$W = \frac{1}{2}mv^2 \rightarrow (2)$

From (1) and (2)  $V = \sqrt{\frac{2GM}{R}}$

35. Orbital speed,  $V = \sqrt{\frac{GM}{R+h}}$

Kinetic energy =  $\frac{1}{2}mv^2$

$\Rightarrow \frac{KE_1}{KE_2} = \left[ \frac{V_1}{V_2} \right]^2 = \frac{R+h_2}{R+h_1} = \frac{R+3R}{R+R} = \frac{4R}{2R} = \frac{2}{1}$

36. Total energy of a satellite,  $E = \frac{-GMm}{2r}$

Work done in shifting,  $W = E_f - E_i$

$W = + \frac{GMm}{2R} \left[ \frac{1}{2} - \frac{1}{3} \right]$

$W = + \frac{gRm}{12}$

$W = \frac{10 \times 6400 \times 10^3 \times 2 \times 10^3}{12}$

$W = 1.066 \times 10^{10} J$

37. Orbital speed  $V_0 = \sqrt{\frac{GM}{R+h}}$

Given  $V_0 = \frac{1}{2}V_e = \frac{1}{2} \sqrt{\frac{2GM}{R}}$

$\Rightarrow \sqrt{\frac{GM}{R+h}} = \frac{1}{2} \sqrt{\frac{2GM}{R}}$

$\Rightarrow \frac{1}{R+h} = \frac{1}{2R}$

$\Rightarrow h = r = 6400 Km$

38. Time period,  $T^2 = \frac{4\pi^2 R_2^3}{GM}$

Acceleration due to gravity

$g = \frac{GM}{R_1^2}$

$GM = \frac{4\pi^2 R_2^3}{T^2}$

2ut

$\Rightarrow g = \frac{4\pi^2 R_2^3}{R_1^2 T^2}$

39. From conservation of angular momentum

$$L_a = L_p$$

$$mr_1v_1 = mr_2v_2$$

$$V_1 = \frac{r_2v_2}{r_1} = \frac{1.5 \times 10^7 \times 5 \times 10^3}{0.5 \times 10^7}$$

$$1.5 \times 10^4 \text{ m/s}$$

40. For smaller heights,

$$g_n = g \left[ 1 - \frac{2h}{R} \right]$$

$$\text{Change in } g, \Delta g_1 = g - g_n$$

$$= \frac{2h}{R}$$

At depth d,

$$g_d = g \left[ 1 - \frac{d}{R} \right]$$

$$\text{Change in } g, \Delta g_2 = g - g_d$$

$$= \frac{d}{R}$$

$$\Delta g_1 = \Delta g_2$$

$$\text{Given } \frac{2h}{R} = \frac{d}{R}$$

$$\Rightarrow d = 2h$$

41.

$$\frac{GM_e}{x^2} = \frac{GM_m}{(d-x)^2}$$

$$\frac{81M_m}{x^2} = \frac{M_m}{(d-x)^2}$$

$$\frac{9}{x} = \frac{1}{d-x}$$

$$x = \frac{9d}{10}$$

42. Potential at a distance R

$$V_p = -\frac{GM}{\sqrt{R^2 + R^2}} = -\frac{GM}{\sqrt{2}R}$$

Potential at the centre

$$V_c = -\frac{GM}{R}$$

$$\text{Work done } W = -[V_p - V_c]m \rightarrow (1)$$

$$\text{And } W = \frac{1}{2}mv^2 \rightarrow (2)$$

$$\text{From (1) and (2), } V = \sqrt{\frac{2GM}{R} \left[ 1 - \frac{1}{\sqrt{2}} \right]}$$



43. Work done on a particle is zero, if force and displacement are perpendicular.

44. Due to 90 Kg, at 'p'

$$E_1 = \frac{G(90)}{3^2} = 10G$$

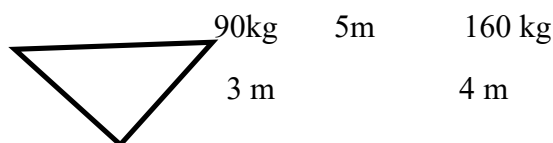
Due to 160 Kg,

$$E_2 = \frac{G(160)}{4^2} = 10G$$

Resultant field,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos 90^\circ}$$

$$E = 10\sqrt{2}GNKg^{-1}$$



45. At a height 'h'

$$g_n = \frac{g}{\left[1 + \frac{h}{R}\right]^2} ; g_h = \frac{g}{4}$$

$$\Rightarrow 4 = \left[1 + \frac{h}{R}\right]^2$$

$$\Rightarrow 2 = 1 + \frac{h}{R}$$

$$\Rightarrow h = R = 6400Km$$

46. In rotational equilibrium

Torque due to

One kid about fulcrum = Torque due to other kid about fulcrum

$$m_1gr_1 = m_2gr_2$$

$$10 \times 2.5 = 15 \times x$$

$$x = \frac{2.5 \times 2}{3} = \frac{5}{3} = 1.67m$$

47. From parallel axes theorem

$$I = I_0 + Mr^2 ; r = \frac{R}{4}$$

$$I = \frac{MR^2}{2} + \frac{MR^2}{16}$$

$$I = \frac{9MR^2}{16} = \frac{9MR^2}{a}$$

48. acceleration,  $a = \frac{10F}{7F}$

$$= \frac{10 \times 490}{7 \times 5}$$

$$= 140ms^{-2}$$

49. From conservation of energy

$$\frac{1}{2}mv^2 \left[ 1 + \frac{K^2}{R^2} \right] = mgh$$

$$h = \frac{V^2 \left[ 1 + \frac{K^2}{R^2} \right]}{2g} = \frac{4 \left[ 1 + \frac{2}{5} \right]}{20} = \frac{7}{5}$$

$$\sin \theta = \frac{h}{1}$$

Distance,

$$\begin{aligned} l &= \frac{h}{\sin \theta} \\ &= \frac{h}{\sin 30^\circ} \\ &= \frac{7}{25} \times \frac{2}{1} \\ &= \frac{14}{25} \\ &= 0.56m \end{aligned}$$

50. Angular momentum is constantss

$$K.E = \frac{L^2}{2I}$$

$$K.E \propto \frac{1}{I}$$

$$\frac{KE_1}{KE_2} = \frac{I_2}{I_1} = \frac{2I_1}{I_1}$$

$$\begin{aligned} KE_2 &= \frac{KE_1}{2} \\ &= 0.5KE_1 \end{aligned}$$